



**Semester 2 Examination, 2019**

**Question/Answer booklet**

**MATHEMATICS  
METHODS  
UNITS 1 AND 2  
Section One:  
Calculator-free**

Fix student label here

Student name: \_\_\_\_\_

**Time allowed for this section**

Reading time before commencing work: five minutes

Working time: fifty minutes

**Materials required/recommended for this section**

***To be provided by the supervisor***

This Question/Answer booklet

Formula sheet

***To be provided by the candidate***

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener,  
correction fluid/tape, eraser, ruler, highlighters

Special items: nil

**Important note to candidates**

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

## Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	8	8	50	52	35
Section Two: Calculator-assumed	13	13	100	98	65
<b>Total</b>					100

## Instructions to candidates

1. The rules of conduct of Christ Church Grammar School assessments are detailed in the Reporting and Assessment Policy. Sitting this examination implies that you agree to abide by these rules.
2. Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.
3. You must be careful to confine your answer to the specific question asked and to follow any instructions that are specified to a particular question.
4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.
5. It is recommended that you do not use pencil, except in diagrams.
6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.
7. The Formula sheet is not to be handed in with your Question/Answer booklet.

**Section One: Calculator-free****35% (52 Marks)**

This section has **eight (8)** questions. Answer **all** questions. Write your answers in the spaces provided.

Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

Working time: 50 minutes.

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**Question 1****(5 marks)**

Determine the gradient of the curve  $y = x^2 - 4x - 60$  at the point(s) where it crosses the  $x$ -axis.

**Question 2****(4 marks)**

The line segment between the points  $A(-1, -2)$  and  $B(-1, 8)$  is the diameter of a circle.

Determine the equation of the circle in the form  $x^2 + ax + y^2 + by = c$ , where  $a, b$  and  $c$  are constants.

**Question 3**

**(9 marks)**

(a) Solve the following exponential equations.

(i)  $25^x = \frac{\sqrt{5}}{125}$

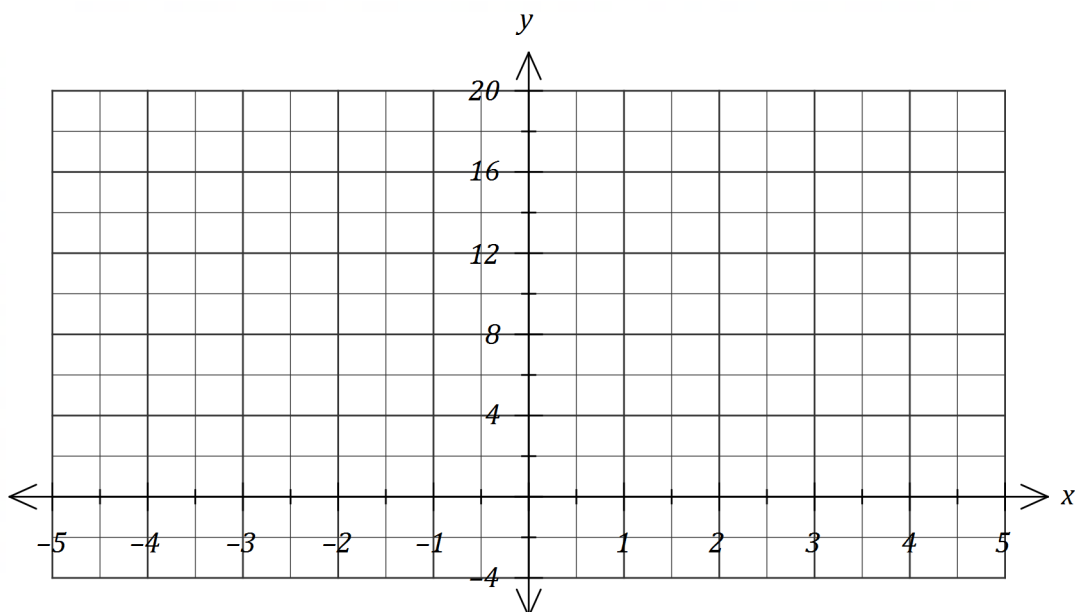
(2 marks)

(ii)  $\sqrt{9x^2+1} = 3^{x+3}$

(4 marks)

(b) Sketch the graph of  $y = 2^{(3-x)}$  on the axes below.

(3 marks)



**Question 4****(7 marks)**

(a) A small body  $A$  is moving along a straight line so that at any time  $t$  seconds, its displacement relative to a fixed point  $O$  on the line is given by  $x = 2t^3 - 9t^2 + 1$  cm.

(i) Determine the velocity of  $A$  when  $t = 1$ . (2 marks)

(ii) Determine the displacement of  $A$  relative to  $O$  at the instant(s) that it is stationary. (3 marks)

(b) A small body  $B$  has velocity given by  $v = 6t^2 - 4t - 2$  cm/s and when  $t = 2$  it has a displacement of 6 cm relative to  $O$ .

Determine an expression for the displacement of  $B$  relative to  $O$  at any time  $t$ . (2 marks)

**Question 5**

**(8 marks)**

(a) Solve the following equations.

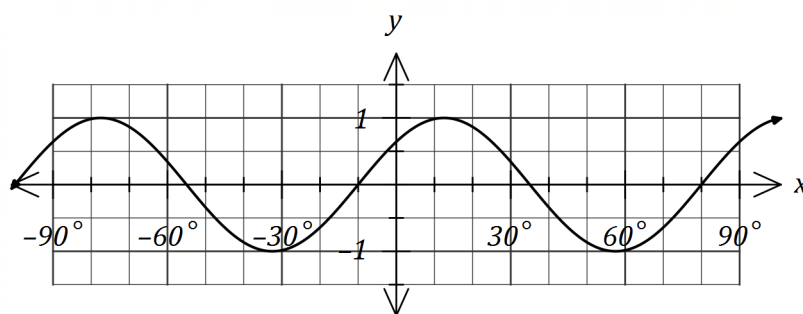
(i)  $\tan(2x) = -\sqrt{3}, 0 \leq x \leq \pi.$

**(2 marks)**

(ii)  $2 \cos(x - 60^\circ) = \sqrt{3} + \cos x, 0^\circ \leq x \leq 360^\circ.$

**(4 marks)**

(b) The graph of  $y = \sin(ax + b)$  is shown below, where  $a$  and  $b$  are positive constants.



Determine the minimum possible value of each of the constants.

**(2 marks)**

**Question 6****(6 marks)**(a) Expand  $(x - 3)^3$ .

(2 marks)

(b) Hence, or otherwise, determine the equation of the tangent to the curve  $y = (x - 3)^3$  at the point where  $x = -1$ . (4 marks)



**Question 7****(6 marks)**

Determine the coordinates of all stationary points of the curve  $y = x^4 + 4x^2 - 12x + 20$ .

**Question 8****(7 marks)**

An arithmetic sequence has a recursive definition given by  $T_{n+1} = T_n + d$ ,  $T_1 = a$ . It has fourth term of 50 and tenth term of 20.

(a) Determine the value of the constant  $a$  and the constant  $d$ . (2 marks)

(b) Determine  $T_{2019}$ . (2 marks)

(c) The sum of the first  $m$  terms of the sequence is 350. Determine the value(s) of the integer constant  $m$ . (3 marks)

Supplementary page

Question number: \_\_\_\_\_

Supplementary page

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**Semester 2 Examination, 2019**

**Question/Answer booklet**

**MATHEMATICS  
METHODS  
UNITS 1 AND 2  
Section Two:  
Calculator-assumed**

Fix student label here

Student name: \_\_\_\_\_

**Time allowed for this section**

Reading time before commencing work: ten minutes  
Working time: one hundred minutes

**Materials required/recommended for this section**

***To be provided by the supervisor***

This Question/Answer booklet  
Formula sheet (retained from Section One)

***To be provided by the candidate***

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators approved for use in this examination

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**Section Two: Calculator-assumed****65% (98 Marks)**

This section has **thirteen (13)** questions. Answer **all** questions. Write your answers in the spaces provided.

Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

Working time: 100 minutes.

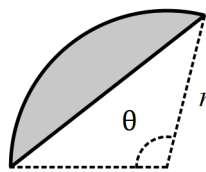
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**Question 9****(6 marks)**

(a) Convert  $108^\circ$  to an exact radian measure.

(1 mark)

(b) A segment of a circle of radius 28 cm is shown below, where  $\theta = 108^\circ$ .



(i) Determine the area of the segment.

(2 marks)

(ii) Determine the perimeter of the segment.

(3 marks)

**Question 10****(7 marks)**

A drone is flying in a straight line and at a constant height  $h$  m above a level pitch towards a thin goal post. It maintains a constant speed of  $4.5 \text{ ms}^{-1}$ .

Initially, the angle of depression from the drone to the base of the post is  $8^\circ$ . Exactly 3 seconds later this angle has increased to  $10^\circ$ .

- (a) Sketch a diagram to show the two angles of depression from the drone to the base of the post. (1 mark)
- (b) Determine, showing all working, the value of  $h$  and calculate the time after leaving its initial position that the drone will collide with the post. (6 marks)

**Question 11**

**(8 marks)**

From a random survey of telephone usage in 320 households it was found that 48 households had access to a mobile phone but not a landline, 268 households had access to a landline and 188 more households had access to a mobile phone than did not.

- (a) Complete the missing entries in the table below. (3 marks)

	Mobile	No mobile	Total
Landline			
No landline	48		
Total			320

- (b) If one household is randomly selected from those surveyed, determine the probability that

(i) it had access to a landline. (1 mark)

(ii) it had no access to a mobile phone given that it had access to a landline. (1 mark)

(iii) it had access to a landline given that it no access to a mobile phone. (1 mark)

- (c) Comment on the possible independence of households having access to a mobile phone and households having access to a landline. Justify your comment. (2 marks)

**Question 12****(10 marks)**

When a manufacturer makes  $x$  litres of a chemical using process  $X$ , the cost in dollars per litre  $C(x)$  varies according to the rule

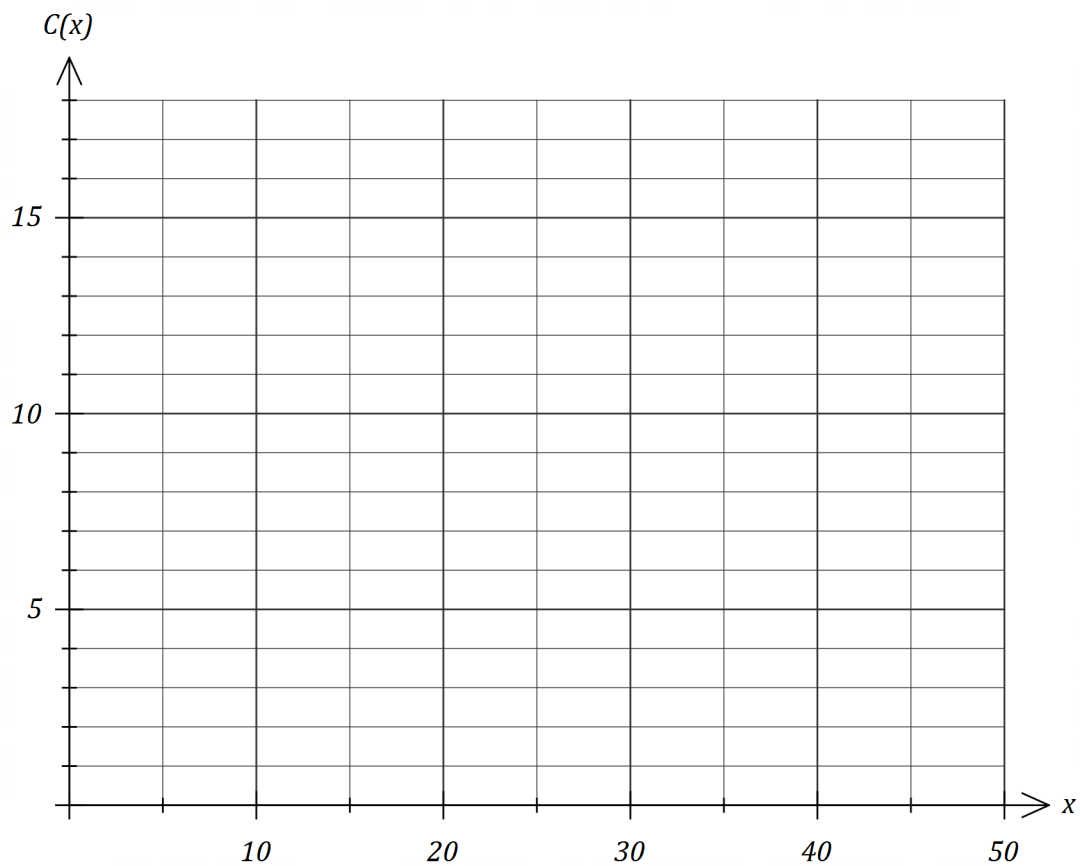
$$C(x) = \frac{240}{x + 15}, \quad 5 \leq x \leq 45.$$

(a) Determine

(i) the cost per litre when 35 L is made. (1 mark)

(ii) the total cost of making 17 L of the chemical. (2 marks)

(b) Graph the cost per litre over the given domain on the axes below. (3 marks)



(c) State the range of  $C(x)$ . (1 mark)

(d) When the manufacturer uses process Z, the cost in dollars per litre  $K(x)$  is modelled by

$$K(x) = 10.5 - \frac{x}{6}, \quad 5 \leq x \leq 45.$$

(i) Add this function  $K(x)$  to the graph. (1 mark)

(ii) determine the production quantities for which process X is cheaper than process Z. (2 marks)

**Question 13****(5 marks)**

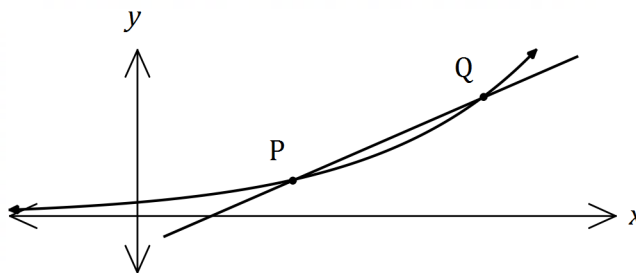
A geometric sequence has a second term of  $-8.4$  and a sum to infinity of  $15$ .

Determine the sum of the first 4 terms of the sequence.

**Question 14**

**(7 marks)**

The graph of  $y = f(x)$  is shown below, where  $f(x) = 4^x$ , together with the secant to the curve through the points  $P$  and  $Q$ .



$P$  has coordinates  $(1, 4)$  and  $Q$  has coordinates  $(1 + h, f(1 + h))$  where  $0 < h \leq 1$ .

- (a) Complete the second column in the table below, rounding values to 4 decimal places where necessary. (4 marks)

$h$	$\frac{f(1 + h) - f(1)}{h}$
1	
0.1	
0.01	
0.001	

- (b) Name the feature of the graph above that the values you calculated in part (a) represent. (1 mark)

- (c) Determine an estimate, correct to 3 decimal places, for the value that  $\frac{f(1 + h) - f(1)}{h}$  approaches as  $h$  becomes closer and closer to 0 and state what this value represents. (2 marks)



**Question 15**

**(7 marks)**

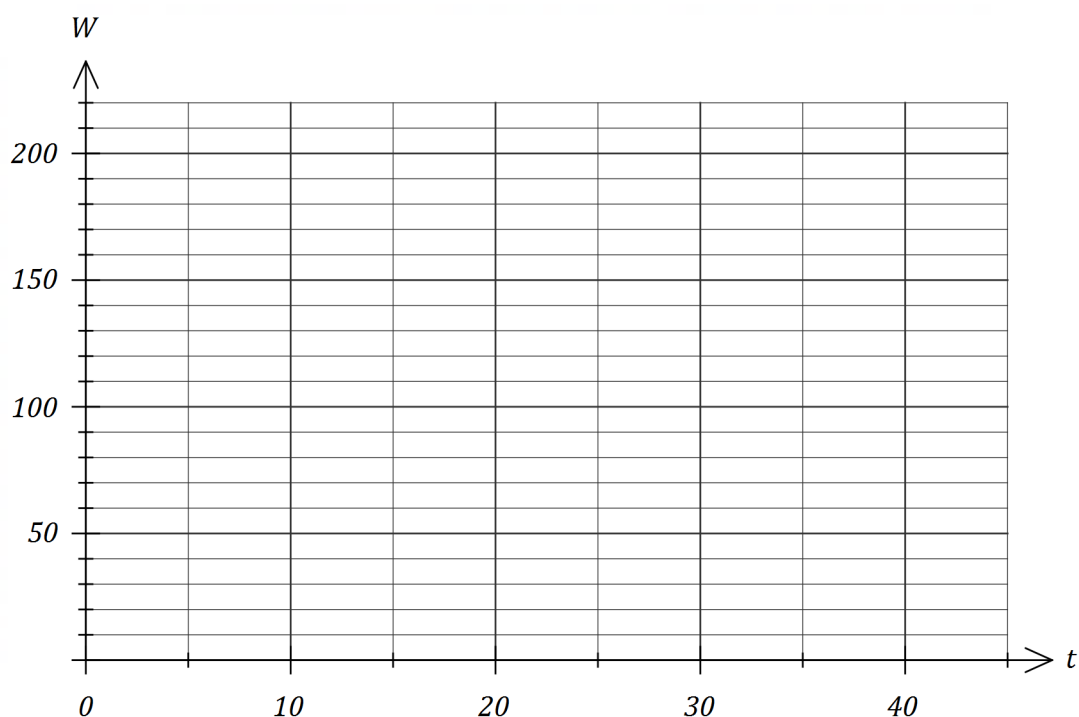
The amount of water in a tank,  $W$  litres, varies with time  $t$ , in minutes, and can be modelled by the equation  $W = 200 - 185(1.2)^{-t}$ ,  $t \geq 0$ .

(a) Determine amount of water in the tank

(i) initially. (1 mark)

(ii) after 15 minutes. (1 mark)

(b) Graph  $W$  against  $t$  for  $0 \leq t \leq 45$  on the axes below. (3 marks)



(c) Over time, the amount of water in the tank approaches  $v$  litres. State the value of  $v$  and determine the time at which the amount of water in the tank reaches 99% of this value. (2 marks)

**Question 16****(7 marks)**

When a patient takes a painkilling drug  $A$ , the probability that they experience some side effects is known to be 0.1.

- (a) A doctor prescribes drug  $A$  to two unrelated patients. Determine the probability that
- (i) neither patient experiences some side effects. (1 mark)
  
  
  
  
  
  
  
  
  
  
  - (ii) one patient experiences some side effects and the other does not. (2 marks)

Other painkilling drugs are available. Of those who take drug  $A$ , 88% of patients who suffer some side effects will switch to another drug whereas no patient who has no side effects will switch.

- (b) The doctor prescribes drug  $A$  to a patient. Determine the probability that the patient does not switch to another drug. (2 marks)
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- (c) The doctor prescribes drug  $A$  to three unrelated patients. Determine the probability that at least one of these patients switch to another drug. (2 marks)

**Question 17****(12 marks)**

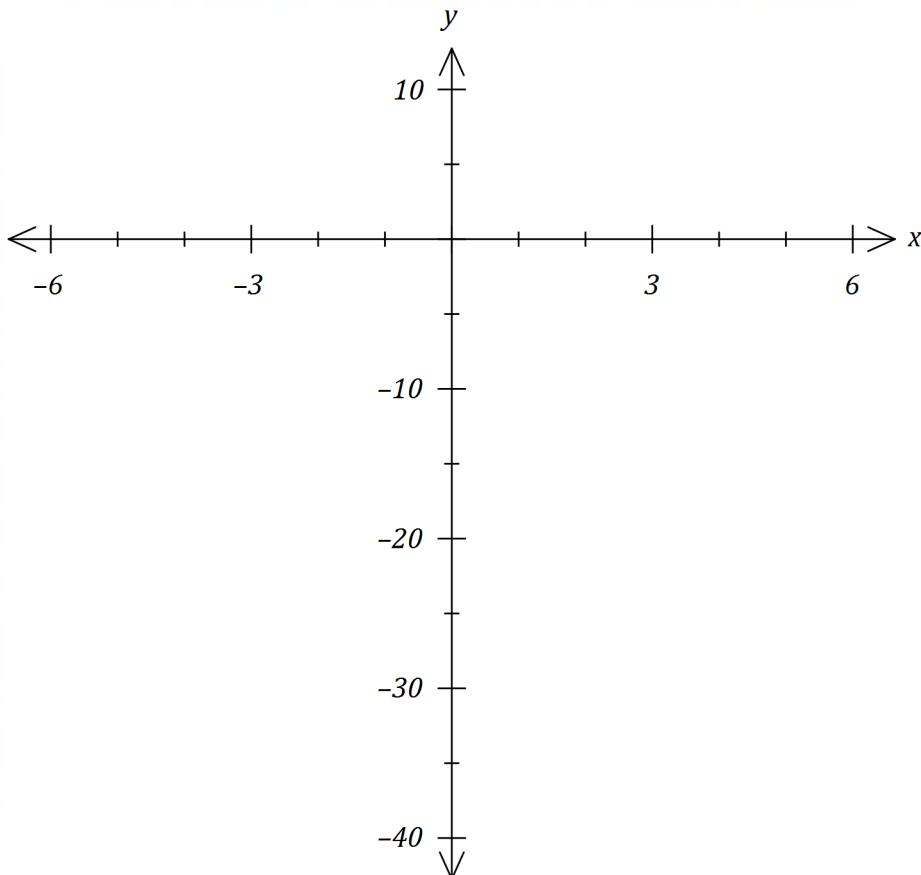
The function  $f$  is defined by  $f(x) = x^3 + ax^2 + bx + c$ , where  $a, b$  and  $c$  are constants.

The graph of  $y = f(x)$  has the following features:

- passes through  $(0, -27)$  and  $(3, 0)$
- has a local maximum at  $(-3, 0)$

(a) Determine the value of  $a$ , the value of  $b$  and the value of  $c$ . (3 marks)

(b) Sketch the graph of  $y = f(x)$  on the axes below. (3 marks)

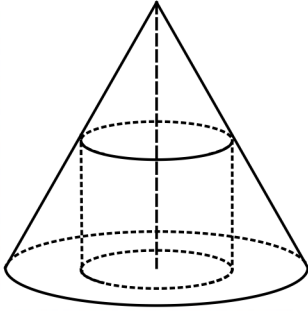


- (c) Use a calculus method to determine the exact coordinates of the local minimum of the graph of  $y = f(x)$ . (3 marks)

- (d) Determine the coordinates of the point where the tangent to  $y = f(x)$  at  $(0, -27)$  intersects the curve  $y = f(x)$ , other than at the point of tangency. (3 marks)

**Question 18****(7 marks)**

A right circular cone of base radius 10 cm and height 25 cm stands on a horizontal surface. A cylinder of radius  $x$  cm and volume  $V$  cm<sup>3</sup> stands inside the cone with its axis coincident with that of the cone and such that the cylinder touches the curved surface of the cone as shown.



- (a) Show that the volume of the cylinder,  $V = 25\pi x^2 - 2.5\pi x^3$ . (3 marks)

- (b) Given that  $x$  can vary, use a calculus method to determine the maximum value of  $V$ . (4 marks)

**Question 19****(8 marks)**

Two events  $A$  and  $B$  are such that  $P(A \cap \bar{B}) = x$ ,  $P(A) = 0.2$  and  $P(\bar{A} \cap B) = 0.6$ .

(a) Determine  $P(A \cap B)$  when  $x = 0.12$ . (2 marks)

(b) Determine an expression for  $P(A \cap B)$  in terms of  $x$ . (1 mark)

(c) Determine the value of  $x$  when

(i)  $A$  and  $B$  are mutually exclusive. (1 mark)

(ii)  $A$  and  $B$  are independent. (2 marks)

(iii)  $P(A|B) = 0.04$ . (2 marks)

**Question 20****(8 marks)**

A fair six-sided dice numbered 1, 2, 3, 4, 5 and 6 is thrown  $n$  times until it lands on a 6.

(a) Show that the probability that  $n = 3$  is  $\frac{25}{216}$ . (1 mark)

(b) Determine the probability that  $n = 5$ . (1 mark)

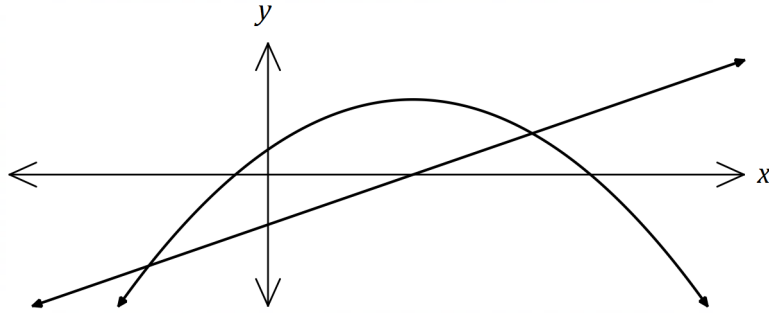
(c) Write an expression in terms of  $n$  for the probability that the first 6 is thrown on the  $n^{\text{th}}$  throw and explain why the probabilities form a geometric sequence. (2 marks)

(d) Determine the probability that the first 6 is thrown in 12 or less attempts. (2 marks)

(e) The probability that the first 6 is thrown in  $k$  or less attempts must be at least 99%. Determine the least value of integer  $k$ . (2 marks)

**Question 21****(6 marks)**

The graphs of  $y = f(x)$  and  $y = g(x)$  are shown below where  $f(x) = 1 + 4x - 2x^2$  and  $g(x) = 2x + k$ .



Determine the value(s) of the constant  $k$  so that the equation  $f(x) = g(x)$  has

(a) one solution.

(5 marks)

(b) no solutions.

(1 mark)



Supplementary page

Question number: \_\_\_\_\_

Supplementary page

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Supplementary page

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